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Speed Trap Optimal Patrolling: STOP Playing Stackelberg Security Games

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Abstract During 2015, 35,092 people died in motor vehicle crashes on the U.S. roadways, an increase from 32,744 in 2014. The 7.2% increase is the largest percentage increase in nearly 50 years. To reduce reckless driving and the resulting accidents, law enforcement agencies deploy speed traps. However, limited resources prevent full coverage at all times, which leaves many roads uncovered. Law enforcement agencies cannot rely on deterministic coverage as it allows drivers to observe and anticipate covered areas. Therefore, randomized speed trap deployment is vital for active road security. This paper provides random and optimal speed traps deployment based on our innovative STOP framework. STOP utilizes game theory to model drivers' and law enforcers' behaviors. In particular, we provide distinct weights to different actions based on the accidents probability, derive the Nash Equilibrium and Stackelberg Security Equilibrium, and determine the best strategies to deploy. The optimal game solution maximizes law enforcer utility, consequently minimizing the cost paid by the society in terms of reducing vehicle accidents.

Keywords Stackelberg games · Accidents probability · Game theory · Active road security

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1 Introduction

Achieving traffic safety is a challenging task for the police around the world. Traffic collisions, fatalities and injuries result in a very high societal cost every year [1–3]. The US National Highway Traffic Safety Administration (NHTSA) statistics show that most road accidents occur with passenger cars (36%) [1]. These alarming statistics draw attention to the road safety issue and underline the urgent need to find a solution that reduces road collisions.

Speed trap deployment is the conventional solution to make the drivers adhere to traffic laws through tickets and fines. Current speed trap deployments, however, face several problems:

- They lack randomization which allows drivers to observe speed trap arrangements over time and form a predictable pattern to their benefits;
- They have limited number of resources which makes it impossible to fully cover the roads at the same time. Thus, speed traps deployment is typically limited to certain roads, leaving many others uncovered;
- They are not optimal as they do not take into consideration the statistics of accidents per roads.

This paper addresses the highlighted issues of speed trap deployments by presenting an innovative platform speed trap optimal patrolling (STOP): STOP is developed to assist law enforcement agencies in scheduling randomized and optimized traffic patrols on their road networks. We also provide an application of STOP in order to help the internal security forces (ISF) in deploying speed traps.

STOP achieves optimality by ensuring maximal road coverage to reduce the road accidents probability and the resulting cost paid by the government. We model the problem a Stacklberg security game (SSG) where the players are the law enforcer and the driver. STOP outputs a probability distribution over the strategies which the law enforcer can use to schedule the speed traps.

The paper is organized as follows: In the second section, we present a literature review on law enforcement solutions. In the third section, we present our platform STOP along with its modules. In the last section, we provide a numerical application of STOP. Finally, we present a result analysis before concluding the paper.

2 Literature Survey

Law enforcement has received wide research interest in the past years. Typically, researchers formulate the problem of deploying law enforcement checkpoints using game theory, mainly Stackelberg game [4–9].

Game theory is a mathematical tool that allows modeling competitive situations where rational decision makers interact to achieve their objectives [10, 11]. A SSG [12] is a two-player game in which a player assumes the role of a "leader" and another assumes the role of a "follower". The SSG takes places in rounds; the leader chooses an action from a set A_1 and the follower responds with an action from a set A_2, after being informed of the leader's choice. The leader in a Stackelberg Security Games is the defender who has to protect a set of targets from the follower (identified as the adversary). The defender employs a finite number of k resources $R = \{r_1, r_2, \ldots, r_k\}$ to protect the set of N targets $T = \{t_1, t_2, \ldots, t_N\}$ against the adversary, such that $k < N$ (resources cannot cover all the targets).

If the players move simultaneously, the standard solution concept is Nash Equilibrium (NE) which is attained when the players have no reason to deviate from the given strategy profile. However, in the Stackelberg model, the defender chooses a mixed strategy first, and the attacker chooses a strategy after observing the defender's choice. In this case, the standard solution is a Strong Stackelberg Equilibrium (SSE), [13]. The SSE is attained when the defender chooses a strategy that maximizes its utility for all possible attacker actions. The attacker responds with a strategy that maximizes its utility given the defender's strategy.

Many models have been developed to address the problem of randomizing the deployment of law enforcement assets [14–17]. These models include ARMOR [18], IRIS [19], PROTECT [20], GUARDS [21], TRUSTS [22]. Nevertheless, most of these models focus on securing infrastructure such as airports, historical landmarks, or a location of political or economic importance, and *none* had considered the problem of traffic patrolling (or deploying speed traps to deter speeding drivers).

In this paper, we present STOP, an innovative patrolling platform. Compared to the existing literature, our contributions are six-fold:

1. We provide a speed trap strategy randomized in space which considers the roads prone to accidents using statistics on accident rates per road.
2. We provide a speed trap strategy randomized in time through temporally randomizing resource allocation; this overcomes drivers' anticipation problem.
3. STOP avoids applying speed traps deployment in congested roads by considering the road congestion situation.
4. We adopt a compact representation of the strategies to minimize the strategies space.
5. We proceed by solving the game using two solutions: NE and SSE. We provide then a comparison between the mentioned solutions to devise the best solution to adopt.
6. To save costs incurred from patrolling shifts, we take into consideration, as part of our cost modeling, the distance traveled by each patrol car.

3 Speed Trap Spatio-Temporal Platform

In this section, we present STOP, the platform that achieves optimal deployment of speed traps in the spatio-temporal domain. It consists of the following six modules as depicted in Fig. 1.

- **Module 1** provides an interface for the law enforcement agency to input the following parameters: date and time, number of available resources, traffic intensity, probability of accidents, and the set of roads.
- **Module 2** outputs uncongested roads that are appropriate for speed traps deployment. It also derives all possible strategies of law enforcer speed traps deployment on selected uncongested roads (section 4).
- **Module 3** computes law enforcers' and drivers' utilities. It provides the payoff matrices needed for equilibrium resolution (section 5).
- **Module 4** computes probability distribution of strategies based on game equilibriums: NE and SSE (section 6).
- **Module 5** outputs a schedule to be implemented by the law enforcer by randomizing strategies over the day (section 7).

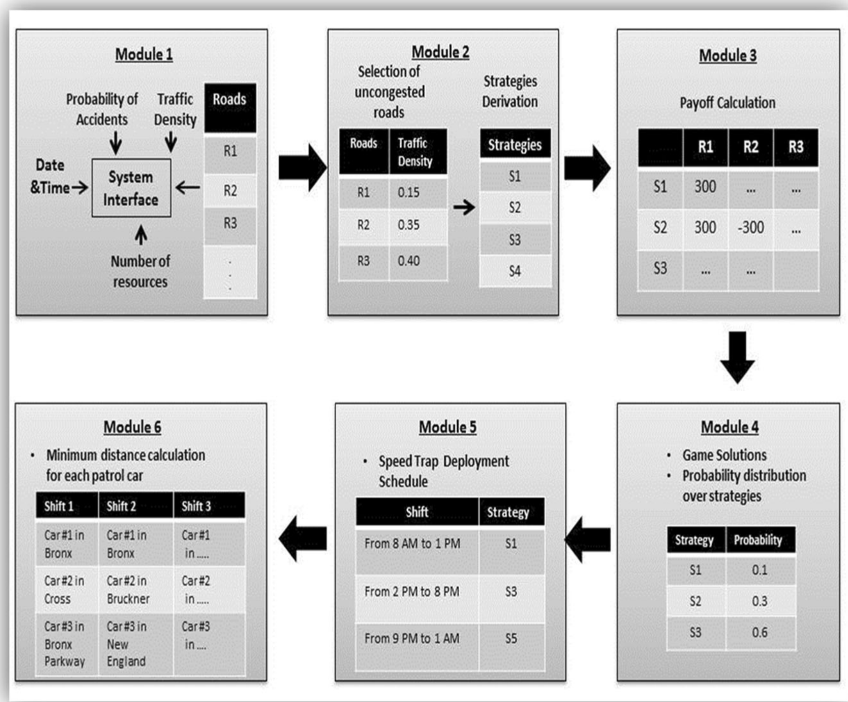


Fig. 1 STOP modules

- **Module 6** computes the minimal distance to be traveled by each patrol car based on the chosen strategy schedule to reduce cost of deployment of the strategy (section 8).

In the rest of this paper, we elaborate on each of STOP's modules and algorithms.

3.1 STOP Module 2: Strategies Derivation

To develop module 2, we proceed in two steps. First, we model a single speed trap with a game between drivers and law enforcers. We then derive the Nash Equilibrium and draw conclusions on how to better apply law enforcement. Second, we extend the study and model multiple speed traps deployment.

3.1.1 Game Model for Speed Trap Allocation

In this section, we focus on a single speed trap scenario. This study will enable us to understand the behavior of a law enforcer and the driver on a single trap and then draw conclusions on the more general case of multiple speed traps. As mentioned before, we model the problem of speed trap deployment as a game with two players: the enforcer and the driver. The enforcer deploys speed traps in a randomized manner, while driver attempts to evade being caught while speeding. We define two strategies for the law enforcer: *applying tough law* or *applying flexible law*.

- A tough enforcer is strict; he deploys a speed trap and spends the whole time working it and patrolling.
- A flexible enforcer, due to limitations in the number of personnel to cover speed traps, will pretend to enforce the law. He will park an uncovered car near the location of a speed trap. This strategy will deceive the driver who would potentially infer that the law is being strictly enforced and thus deter him/her from speeding.

In response, the driver may adopt either one of these two strategies: *violating* or *obeying* the law. The game between the law enforcer and the driver takes place as follows:

- The enforcer chooses a law enforcement strategy t_i, $i \in (1, 2)$ from the feasible set $T = \{t_1, t_2\}$, according to the probability distribution $p(t_i)$, where t_1 represents the tough-strategy and t_2 represents the flexible-strategy. For each i, the following conditions stands: $p(t_i) \geq 0$ and $\sum p(t_i) = 1$.
- The driver observes t_i. Then s/he selects, a_k, ($k \in (1, 2)$) from the feasible action set $A = \{a_1, a_2\}$, where a_1 refers to offending the law while a_2 to obeying the law.
- Once the enforcer is aware of the driver strategy, he selects an action b_n, $n \in (1, 2)$, from $B = \{b_1, b_2\}$, where b_1 (resp. b_2) refers to punishing (resp. not punishing) the driver.
- The enforcer and driver payoffs are respectively $U_e(t_i, a_k, b_n)$ and $U_d(t_i, a_k, b_n)$. Table 1 summarizes players' actions notations.

We assume that the enforcer has knowledge of his/her utility function and that of the driver. On the other hand, the driver knows his/her utility function but has no access to the enforcer's utility function. Before detailing the utility functions, we introduce the following parameters:

- g is the net gain of the driver defined as savings in travel time by the driver (potentially through speeding).
- s is the punitive cost to the driver which is the speeding fine.
- λ is the parameter mapping the punishment into the negative utility.
- G is the social welfare.
- r_1 is the law credibility in case of covered speed trap.
- r_2 is the law credibility in case of uncovered speed trap.

Figure 2 exhibits the extensive form game. In case the enforcer is tough (t_1), the driver chooses between:

- *Violating* (a_1) where s/he will receive a penalty from the law enforcement and consequently a negative utility $U_d(t_1, a_1, b_1) = \mathrm{g} - \lambda\,\mathrm{s} \leq 0$. It is worth noting that g −

Table 1 Notation list

Notation	Description
a_1	The driver chooses violation
a_2	The driver obeys the law
t_1	The enforcer is tough
t_2	The enforcer is flexible
b_1	The enforcer is punishing the driver
b_2	The enforcer is not punishing the driver

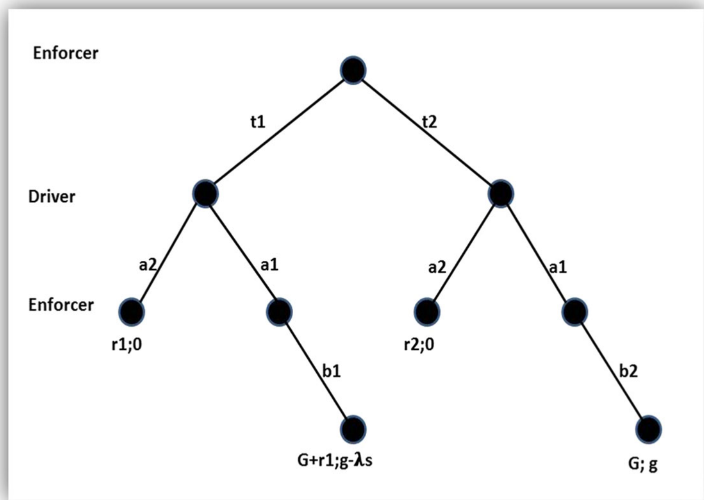


Fig. 2 The extensive game form

λ s should be negative, otherwise the punishment does not make sense. In such case, the enforcer will receive a positive utility equal to $U_e(t_1, a_1, b_1) = G + r_1$.

- *Obeying the law* (a_2) then s/he will receive a utility $U_d(t_1, a_2, b_2) = 0$, while the enforcer will receive a positive utility in the form of credibility $U_e(t_1, a_2, b_2) = r_1$.

Alternatively, if the enforcer chooses the flexible strategy (t_2), then the driver can decide between:

- *Violating* (a_1): the driver will not be punished and will receive a utility of $\mathrm{U_d(t_2, a_1, b_2)} = g \geq 0$, while the enforcer will receive 0 utility such that $U_e(t_2, a_1, b_2) = 0$.
  It is to be noted that the proposed framework strives to enhance active road security with a limited number of resources. In this context, we assume that a flexible enforcer, contrarily to the tough enforcer, adopts a smooth strategy. More precisely, the flexible enforcer places an *uncovered* car to dupe the driver and thus force him to decelerate. This is achieved while minimizing the cost of speed traps and increasing the law credibility.
- *Obeying the law* (a_2) the driver will receive zero utility ($U_d(t_2, a_2, b_2) = 0$), while the enforcer will receive a positive utility in the form of law enforcement credibility: $U_e(t_2, a_2, b_2) = r_2$, such that $r_2 < r_1$.

At this stage, we proceed with computing the Nash Equilibrium. We denote by p the probability of the law enforcer choosing t_1 and by $(1 - p)$ the probability of the law enforcer choosing t_2. Table 2 shows the payoff matrix.

The Nash Equilibrium is achieved by the following equation:

$$p(g - \lambda s) + (1 - p)g = 0 \tag{1}$$

Table 2 Enforcer and driver payoff matrix

Enforcer/driver	a_1	a_2
t_1	$G + r_1;\ g - \lambda s$	r_1; 0
t_2	$0;\ g$	r_2; 0

We calculate the probability p and $(1 - p)$ so that the driver will be indifferent in terms of the utility if s/he chooses the actions a_1 or a_2, regardless of the enforcer's action. As a result, we obtain $p = \frac{g}{\lambda s}$; this value of p governs the enforcer's mixed strategy (choosing between t_1 or t_2). With this mixed strategy, the driver has no incentive to speed, as the expected utility of violating (a_1) or obeying (a_2) the law are the same.

In the following sections, we extend this study to a multiple speed trap scenario. We model the problem with a SSG where the leader is the law enforcer and the follower is the driver.

3.1.2 Game Model for Speed Trap Allocation

A law enforcement agency has a limited number of available resources. Therefore, to reduce car accidents and optimally allocate the limited speed traps, we should adopt a dynamic and randomized strategy accounting for temporal parameters, accidents probability, traffic congestion percentage and number of resources.

To achieve this randomization, we model the decision of deploying the speed traps as an SSG, where the leader is the law enforcer and the follower is the driver. The law enforcer covers a subset of the roads segments using the available (albeit limited) number of speed traps and the driver chooses the segment where to violate.

The law enforcer employs a mixed strategy to mislead the driver of the exact place of the speed traps. The possible law enforcer's actions are the set of possible speed traps combinations. For example, if we have to set 2 radars on 3 roads segments A, B and C, therefore we will have C_3^2 possible strategies for the ISF which are: covering (A, B), covering (A, C) or covering (B, C).In this case, the driver will have to choose between violating segment A, violating segment B or violating segment C.

For the model tractability, we partitioned the highways into road segments. Where speed traps are separated by an average distance calculated according to several factors that are out of the scope of our paper.

3.1.3 Compact Representation

The strategy space of the enforcer and driver is a function of the number of road segments. As such, enumerating all the combinations of strategies presents a scalability challenge to solve for the optimal strategy. We address this issue by reducing the size of the space of the possible strategies as follows:

First, we focus on road segments where the traffic speed could reach more than 30 mph $\simeq$ 50 km/h, those segments exhibit a higher rate of fatal road accidents [23]. Second, we minimize the space of strategies by developing a distributed algorithm to be executed by a local law enforcement agent. A typical law enforcement agency partitions its area of coverage into a set of command centers (each center controlling a region) [24].

Each center runs the STOP platform on its covered region so that STOP manipulates a subset of the number of roads which reduces the strategies space. Third, STOP does not consider congested roads which do not represent speeding opportunities for the drivers.

3.2 STOP Module 3: Payoff Calculation

STOP considers two payoff matrices: one for the law enforcer and another for the driver according to their utility functions. The enforcer's payoff is determined as follows:

- If the driver violates a covered road, then the enforcer receives $G > 0$ where G corresponds to the social welfare.
- If the driver violates an uncovered road, then the enforcer receives a negative utility $-G^*P_a(t)$, where $P_a(t)$ is the accident probability at the considered road t.

Alternatively, the driver payoff is determined as follows:

- If the enforcer is covering a road segment and the driver violates it, then s/he is punished by paying the fine. In such case, the driver's payoff is $g - \lambda s \leq 0$, where g is the driver's gain (the time saved by the driver), s is the punitive cost (the ticket price paid) and λ is a parameter mapping the punishment into the negative utility.
- If the driver violates an unprotected road segment, then s/he receives a positive utility g, which is the saved time.

Table 3 contains an example of the payoff matrix with 2 speed traps deployed on 3 roads segments: A, B and C. The enforcer has only three (C_3^2) possible strategies: protect segments A and B, protect segments A and C or protect segments B and C at the same time. In response, the driver can choose between violating segment A, violating segment B or violating segment C.

3.3 STOP Module 4: Game Equilibrium Solutions

Given the payoff matrices of the enforcer and driver, we proceed into solving the equilibrium solutions. In this section, we aim at comparing two solutions NE and SSE. This will help us to determine the best approach to adopt in STOP.

3.3.1 Nash Equilibrium (NE)

The mixed strategy NE solution is solved by finding the probability distribution over the strategies of each player. The NE is attained if the strategies are mutually best responses to each other, so that the players have no reason to deviate from the given strategy profile. In other words, our objective is to find a probability distribution over the enforcer's strategies in a way that the driver is indifferent from choosing any of his/her strategies. At the same time, we should find the probability distribution over the driver's strategies in a way that

Table 3 Enforcer and driver payoff matrix

Enforcer/driver	A	B	C
A, B	G; g − λs	G; g − λs	− G*Pa(C); g
A, C	G; g − λs	− G*Pa(B); g	G; g − λs
B, C	− G*Pa(A); g	G; g − λs	G; g − λs

the enforcer is indifferent from choosing between his/her strategies. Having n strategies for the enforcer and m strategies for the driver, we define R as the enforcer payoff matrix and C the driver payoff matrix, such that:

- R_{ij} is the enforcer's payoff corresponding to his/her strategy i, $i \in \{1, 2,\ldots,n\}$ and to the driver strategy j, $j \in \{1, 2,\ldots,m\}$.
- C_{ij} is the driver's payoff corresponding to his/her strategy j, $j \in \{1, 2,\ldots,m\}$ and to the enforcer's strategy i, $i \in \{1, 2,\ldots,n\}$.

The mixed strategy NE solution is solved by finding the probability distribution over the strategies of each player. Let the vector $x = \langle x_i \rangle$ be the mixed strategy for the enforcer, where x_i is the probability of the enforcer choosing strategy i, and the vector $q = \langle q_j \rangle$ be the mixed strategy of the driver, where q_j is the probability of the driver choosing strategy j. We can then obtain NE as follows:

For the enforcer, we solve the following set of equations:

$$\begin{cases} \sum_{i=1}^{n} x_i C_{ij} = \sum_{i=1}^{n} x_i C_{i1} \quad \forall j > 1 \\ \sum_{i=1}^{n} x_i = 1 \end{cases} \tag{2}$$

For the driver, we solve the following equations:

$$\begin{cases} \sum_{j=1}^{m} q_j R_{ij} = \sum_{j=1}^{m} q_j R_{1j} \quad \forall i > 1 \\ \sum_{j=1}^{m} q_j = 1 \end{cases} \tag{3}$$

Equation (2) specifies that to find the probabilities x_i, we should ensure that the driver receives the same utility for any strategy j chosen by the enforcer. Similarly, Eq. (3) indicates that we should find the probabilities q_i in a way that the enforcer receives the same utility for any strategy i chosen by the driver. Solving both equations allows us to derive a probability distribution for the driver's and the enforcer's strategies leading to the mixed strategy NE.

3.3.2 Stackelberg Equilibrium

The NE solution guarantees that each player receives the best utility regardless of the other player's strategy. The SSE, on the other hand, attempts to search directly for an optimal enforcer strategy which is more desirable in our context of traffic patrolling and citizens safety.

To solve for SSE, we define a Mixed Integer Quadratic Program (MIQP) and present a linearized equivalent Mixed Integer Linear Program (MILP). First, the driver chooses a strategy that maximizes his/her utility. According to this strategy, we find the enforcer's mixed strategy that provides the highest utility.

We denote by:

- X and Q the index sets of enforcer and driver's pure strategies, respectively.
- $x = \langle x_i \rangle$ the enforcer's mixed strategy vector where x_i is the probability of employing strategy i.

- $q = <q_j>$ the driver's pure strategies vector where $q_j \in \{0, 1\}$, q_j is equal to 1 when the strategy j is employed by the driver.

As specified earlier, the payoff matrices R and C are defined such that R_{ij} represents the enforcer's utility and C_{ij} the follower's utility when the enforcer adopts pure strategy i and the driver applies pure strategy j.

The enforcer's MIQP problem is defined in Eq. (4) as:

$$\begin{aligned}
&\max_{x,q,a} \sum_{i \in X} \sum_{j \in Q} R_{ij} x_i q_j \\
&\text{s.t.} \quad \sum_{i \in X} x_i = 1 \\
&\qquad \sum_{j \in Q} q_j = 1 \\
&0 \leq \left(a - \sum_{i \in X} C_{ij} x_i \right) \leq (1 - q_j) M \\
&x_i \in [0, 1] \\
&q_j \in \{0, 1\} \\
&a \in R
\end{aligned} \tag{4}$$

Our objective is to obtain the mixed strategy of the enforcers that maximizes its expected utility over all possible driver strategies, subject to a set of constraints. The first and fourth constraints define x_i as the probability distribution of strategies. The second and fifth constraints limit the vector q to a pure distribution over the driver's strategies; q_j is equal to one when the driver chooses the pure strategy j, and the remaining indices are equal to zero. The third constraint ensures that $q_j = 1$ for strategy j that is optimal for the driver: the left-side inequality ensures that for all $j \in Q$, $a \geq \sum_{i \in X} C_{ij} x_i$. This means that for a given vector x, a is an upper bound for the driver's utility for any strategy. The right-side inequality is inactive for every action where $q_j = 0$ since M is a large positive quantity. For the action that has $q_j = 1$, this inequality states that the adversary's payoff for this action must be $\geq a$, which combined with the previous inequality shows that this action must be optimal for the driver.

We linearized the previous MIQP through the change of $z_{ij} = x_i q_j$. We obtain the following MILP as stated in Eq. (5):

$$\begin{aligned}
&\max_{q,z,a} \sum_{i \in X} \sum_{j \in Q} R_{ij} z_{ij} \\
&\text{s.t.} \quad \sum_{i \in X} \sum_{j \in Q} z_{ij} = 1 \\
&\qquad \sum_{j \in Q} z_{ij} \leq 1 \\
&\qquad q_j \leq \sum_{i \in X} z_{ij} \leq 1 \\
&\qquad \sum_{j \in Q} q_j = 1 \\
&0 \leq \left(a - \sum_{i \in X} C_{ij} \left(\sum_{h \in Q} z_{ih} \right) \right) \leq (1 - q_j) M \\
&z_{ij} \in [0 \ldots 1] \\
&q_j \in \{0, 1\} \\
&a \in \Re
\end{aligned} \tag{5}$$

3.3.3 NE and SSE Comparison

Table 5 presents an example of a comparative study between the NE and the SSE. In this example, we consider 3 roads: *Road*1, *Road*2 *and Road*3, with 2 available speed traps (similar to Table 3). We have the following payoff matrices for the driver and the enforcer (Table 4).

Our proposal focuses on enhancing road safety which is related to maximizing the enforcer utility. Therefore, we opt for adopting the Stackelberg security game.

In fact, with the Stackelberg game framework, the law enforcer (defender) acts first by committing to a patrolling strategy, and the driver (attacker) chooses where to attack after observing the defender's choice. The typical solution assumes that the defender chooses an optimal mixed strategy based on the assumption that the attacker will observe this strategy and choose an optimal response.

More specifically, the SSE searches directly for the driver's strategy that maximizes the enforcer's utility while the Nash equilibrium (NE) gives a probability distribution over both the enforcer's and driver's strategies. With NE, no player can obtain a higher profit by choosing a different strategy. Consequently, no player (driver or enforcer) wants to change its strategy.

Since we are mainly concerned with the road active security, maximizing the enforcer utility is a goal. Table 5 shows that SSE assigns the enforcer a utility(100) that is higher than that of NE (35.77). Therefore, the Stackleberg security game or the leader–follower paradigm is best suited to model interactions between the security forces and drivers.

3.4 STOP Module 5: Speed Trap Deployment Schedule

According to the obtained probability distribution over the strategies for each shift, the enforcer chooses the best strategy to deploy. The choice of strategies for each shift gives a schedule for the speed traps.

For reader clarity, we detail hereafter the speed trap deployment process on a set of six roads with 3 available speed traps, and 3 shifts: shift1 (from 8 AM to 1 PM), shift2 (from 2 PM to 7 PM) and shift3 (from 8 PM to 1 AM) on Monday 13/2/2017.

Table 4 Enfocer and driver payoff matrices

Enforcer's payoff matrix			
Driver violates→	Road1	Road2	Road3
Road1–Road2	300	300	− 150
Road1–Road3	300	− 90	300
Road2–Road3	− 120	300	300
Driver's payoff matrix			
Driver violates→	Road1	Road2	Road3
Road1–Road2	300	300	− 150
Road1–Road3	300	− 90	300
Road2–Road3	− 120	300	300

Table 5 NE and SSE comparison

	NE	SSE
Enforcer's probability distribution over the strategies	$x_1 = x_2 = x_3 = 1/3$	$x_1 = 0.3333333381900081$; $x_2 = 0.33333332361998375$; $x_3 = 0.33333333819000804$
Driver's probability distribution over the strategies	$q_1 = \frac{195}{587}$; $q_2 = \frac{210}{587}$; $q_3 = \frac{182}{587}$	$q_2 = 1$ and $q_1 = q_3 = 0$
Enforcer's utility in case of choosing strategy 1 for the enforcer and strategy 2 for the driver	$\frac{1}{3} \times \frac{210}{587} \times 300 = 35.77$	$0.3333333381900081 \times 1 \times 300 = 100$

In a first step, STOP platform collects inputs (probability of accidents, traffic density and number of resources) related to Shift 1. Then, STOP derives strategies and computes the game solution as described in modules 1, 2, 3 and 4 of Fig. 1.

The probability distribution is exhibited in Fig. 3. One can see that the maximum probability is associated with the strategy: 'Road2–Road3–Road4'. Consequently, the speed traps are deployed in these roads for shift 1. The adopted steps are repeated and applied for the subsequent shifts.

Table 6 exhibits the speed trap deployment schedule for 3 shifts of Monday and Tuesday.

3.5 STOP Module 6: Minimal Distance Calculation

To reduce the cost incurred by the law enforcement agency, we design a plan that minimizes the distance travelled by a patrolling car between shifts. When a shift ends, the patrol cars should travel to another area to cover it. Therefore, to save the time spent during transportation and reduce pollution, we propose an algorithm that decides on travel plan for a patrolling car during a work day. Given S^h strategies scheduled for each epoch and d the roads distance, the algorithm proceeds as follows:

```
######## Driver's Pure Strategy ########
Action: Road3

######## Leader's Mixed Strategy ########
Probability of action:  Road1 Road2 Road3 : 0.0
Probability of action:  Road1 Road2 Road4 : 0.0
Probability of action:  Road1 Road2 Road6 : 0.19999999417199027
Probability of action:  Road1 Road3 Road4 : 0.0
Probability of action:  Road1 Road3 Road6 : 0.20000000874201454
Probability of action:  Road1 Road4 Road6 : 0.1999999941719903
Probability of action:  Road2 Road3 Road4 : 0.4000000029140048
Probability of action:  Road2 Road3 Road6 : 0.0
Probability of action:  Road2 Road4 Road6 : 0.0
Probability of action:  Road3 Road4 Road6 : 0.0
```

Fig. 3 Probability distribution over strategies obtained for shift 1

Table 6 Schedule example

Lists of roads	Road1	Road2	Road3	Road4	Road5	Road6
Monday 13/2/2017						
Shift1		×	×	×		
Shift2		×		×	×	
Shift3	×		×			×
Tuesday 14/2/2017						
Shift1	×		×		×	
Shift2		×		×		×
Shift3	×		×			×

Algorithm 1: Minimal traveled distance

1. For each time epoch *h*, from *h=H* to *h=2*
1a. Get the strategy S^h for the time epoch *h*
1b. Get the strategy S^{h-1} for the time epoch *h-1*
1c. Solve the Linear Program 1

Linear Program 1

$$\min d$$
$$\text{s.t. } \sum_i s_{i,j}^h = 1, \forall j$$
$$\sum_j s_{i,j}^h = 1, \forall i$$
$$d \geq s_{i,j}^h \, . dist_{S^h(i), S^{h+1}(j)} + D^{h+1}(j), \forall i, j$$
$$s_{i,j}^h \, \epsilon \{0,1\}, \forall i, j$$

The linear program 1's objective is to derive a driving plan for each patrol car to cover all the speed traps and minimize the travel time. Therefore, we oriented our efforts towards elaborating an optimization model that minimizes the maximum distance travelled by each patrolling car. Let $S^h(k)$ denote the k^{th} target in the h^{th} shift. For example, $s_{ij}^h = 1$ indicates that the car that was used in strategy $S^h(i)$ at road i, will be used in $S^{h+1}(j)$ to cover road *j*. $D^h(j)$ denotes the distance traveled from $S^h(j)$ at shift *h* to *H*. The first and second constraints ensure that one target at *h* is connected to only one target at $h + 1$ and vice versa. The third constraint computes the total distance traveled by the patrol teams.

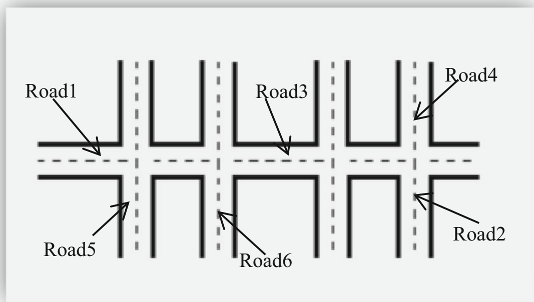
4 Simulation and Results

In this section, we provide numerical results and elaborate conclusions on the optimal driving plan and speed trap deployment utility.

We implemented a simulator to evaluate STOP. It takes as inputs the traffic density per road, accidents probability, number of resources, and the list of roads.

In order to evaluate STOP, we considered a road segments map presented in Fig. 4. We assumed traffic densities and probability of accidents delivered by the National Council for Scientific Research (CNRS) in Lebanon as part of an ongoing national transportation project.

Fig. 4 Road segments map example



We simulated the STOP framework over 3 different scenarios. Scenario 1 evaluates the impact of shifts and resources on enforcer's utility. Scenario 2 studies the probability distribution of strategies with different probability of accidents. Scenario 3 tackles the extreme accidents probability and evaluates the impact of resources on enforcer's utility.

4.1 Scenario1: Study of Enforcer's Utility Variation

Scenario 1 aims at evaluating the impact of deploying number of resources during the different shifts on the enforcer's utility. Scenario 1 parameters are identified in Table 7. Table 7 shows the considered roads, along with their traffic density, the probability of accidents on each road, at each shift and the different number of resources. It is to be noted that the metrics adopted in STOP platform and exhibited in Tables 7 and 8 are provided by the Lebanese National Council of Research.

We evaluate the enforcer's utility for each shift and number of resources as evident in Fig. 5. First, we observe that the enforcer's utility has the lowest values in the first shift of patrolling (from 8 AM to 1 PM); this is due to the high probability of accidents at this interval of time of the day. In the second and third shift of patrolling the probability of accidents starts to decrease and therefore the enforcer's utility is higher than the first shift. It is also clear that the increase in deployed resources on roadways, correspond to an increase in the enforcer's utility.

Table 7 Scenario 1 parameters

List of roads (L)	Road1–Road2–Road3–Road4–Road5–Road6
Traffic density	0.30, 0.35, 0.34, 0.30, 0.35, 0.30, 0.30
Probability of accidents	Shift1 (from 8 AM to 1 PM): 0.38–0.35–0.34–0.37–0.38–0.38–0.35
	Shift2 (from 2 PM to 7 PM): 0.31–0.32–0.31, 0.32–0.29–0.30–0.31
	Shift 3 (from 8 PM to 1 AM): 0.11–0.12–0.11–0.12–0.09–0.10–0.11
Number of resources	2–3–4–5–6

Table 8 Scenario 2 parameters

List of roads (L)	Road1	Road2	Road3	Road4	Road5	Road6
Number of resources	2					
P_1	0.75	0.85	0.80	0.40	0.5	0.5
P_2	0.75	0.25	0.25	0.40	0.6	0.5
Percentage of congestion (%)	30	30	25	30	70	65

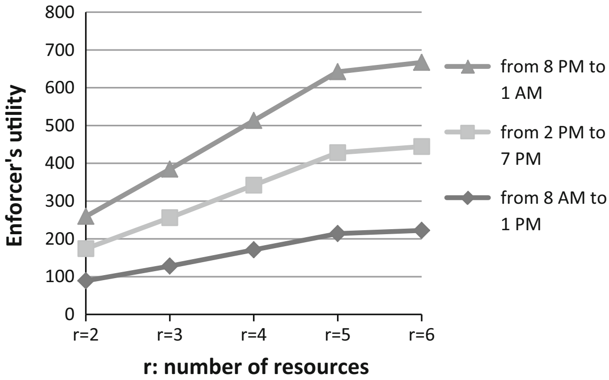
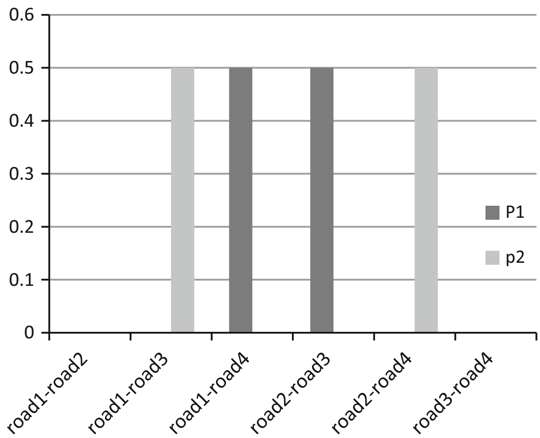


Fig. 5 Enforcer's utility during three different epochs of the day

4.2 Scenario 2: Study of Probability Distribution Over Strategies

In the present scenario, we evaluate the probability distribution over the strategies according to the probabilities of accidents. The goal is to validate the utility of speed traps deployment on accidental roads. Table 8 exhibits the list of adopted parameters related to two sets of accidents probability P_1 and P_2.

Fig. 6 The probability distribution over strategies with various probability of accidents



The bar chart of Fig. 6 shows the variation of the strategies' probability as function of the variation of the probability of accidents on each road. We should note that we considered a percentage of congestion threshold equal to 40%, as a result road5 and road6 are not included in the strategy derivation.

We focus on roads 2 and 3 where we consider high and low values of accident probabilities. In case of a high accident probability on both roads (0.8 and 0.85), we obtained a strategy probability on road2–road3 equal to 0.5. This result confirms the relevance of deploying speed traps on accidental roads. However, when the probability of accidents in road2 and road3 is 0.25, the probability of adopting the strategy road2–road3 is approximately zero.

The probability distribution over the strategies varies with the probability of the accidents on each road. It worth noting that STOP estimates the mixed strategy to give the maximum payoff for the enforcer while ensuring an optimal response for the driver. Therefore, some strategies are not used or have low probabilities even if they appear very important because of the high probability of accidents on the considered roads.

4.3 Scenario 3: Study of Extreme Conditions of Accident Probabilities

In this scenario, we study two extreme cases of the probability of accidents. For each case, we calculate the enforcer's utility according to various number of resources. Table 9 shows the list of considered parameters.

Scenario 3 results are shown in Figs. 7 and 8. In this scenario, we consider two extreme cases, with the probability of accidents (P) equal to either 1 or 0. We compare the enforcer's utility in each case by varying the number of available resources.

When the probability of accidents is 1, we notice in Fig. 7 that when we use 2 resources to cover the 6 considered roads, the enforcer receives a negative utility. In this case, the enforcer is leaving many roads unprotected leaving the drivers to face a high risk of accidents. As the number of resources increases, the utility of the enforcer increases and assumes a positive value. The enforcer will be covering more roads which reduces the risk of drivers having accidents.

In the extreme case of a zero probability of accidents (Fig. 8), the roads are safe and there is no risk of traffic accidents and fatalities. By adding more resources, we observe in Fig. 8 that the enforcer's utility increases. This utility increase is compatible with our linear program that calculates the enforcer's utility according to the enforcer's payoff matrix and it is clear that the more we add resources the less we get negative utility, since when the enforcer is covering a certain road, s/he receives a positive utility. Moreover, the utility increases as the number of resources increases.

Table 9 Scenario 3 parameters

List of roads L	Road1–Road2–Road3–Road4–Road5–Road6					
Number of resources	{2, 3, 4, 5}					
Probability of accidents (P)	{0, 1}					
Traffic density	Road1	Road2	Road3	Road4	Road5	Road6
	0.3	0.3	0.25	0.3	0.3	0.35

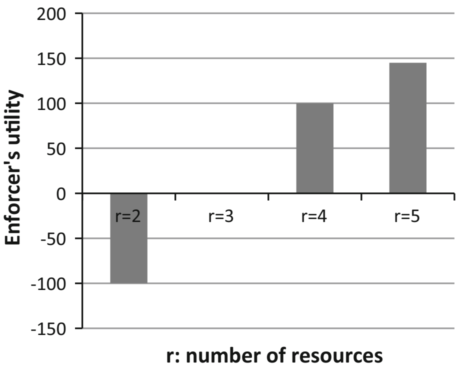


Fig. 7 Defender's utility with different number of resources (2, 3, 4, 5) when P = 1

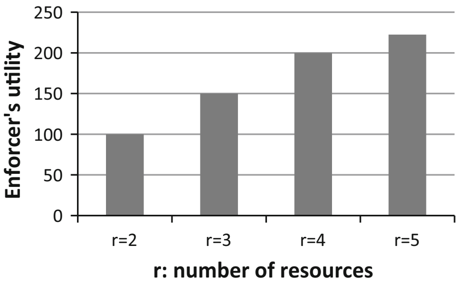


Fig. 8 Defender's utility with various number of resources (2, 3, 4, 5) when P = 0

5 Conclusion

Traffic patrolling is a challenge faced by any law enforcement agency. To avoid drivers predicting traffic patrolling schedules, randomizing these schedules is a must. Moreover, any traffic patrolling schedule should consider a set of parameters including: date and time, probability of accidents on roads, percentage of congestion, and number of available resources.

In this paper, we present STOP, a framework that assists a law enforcement agency in optimally deploying speed traps. We model the interaction between drivers and enforcers using an SSG to find the optimal randomized strategy that ensures the coverage for the maximum number of roads.

We solved the SSG using two approaches: the SSE and NE, and showed that the SSE is more compatible in our context.

We evaluated STOP in three different scenarios. We found that the enforcer's utility increases when the accident probability is reduced and the probability of using a strategy is related to the probability of accidents on the considered roads (the strategies probability is higher on accidental roads).

We plan to follow up on this work by further exploring the notion of flexible law enforcement (Sect. 4.1), especially when the accident probability on certain roads is very low. This could help ensure the law credibility and reduce law enforcement costs.

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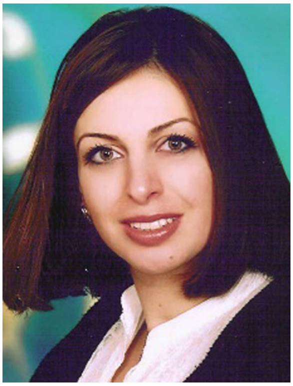

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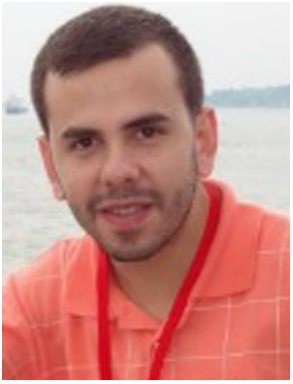

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